Motion in a Straight Line

A Watercolor Study: From Intuitive Flow to Mathematical Precision

Motion is common to everything in the universe—from the blood flowing in arteries to the Earth revolving around the Sun. To understand this flow, we must learn to describe it. In this study, we treat the straight line not as a rigid constraint, but as a focused stroke of the brush. We will move from the concept of position to the precision of calculus, painting a complete picture of Rectilinear Motion.

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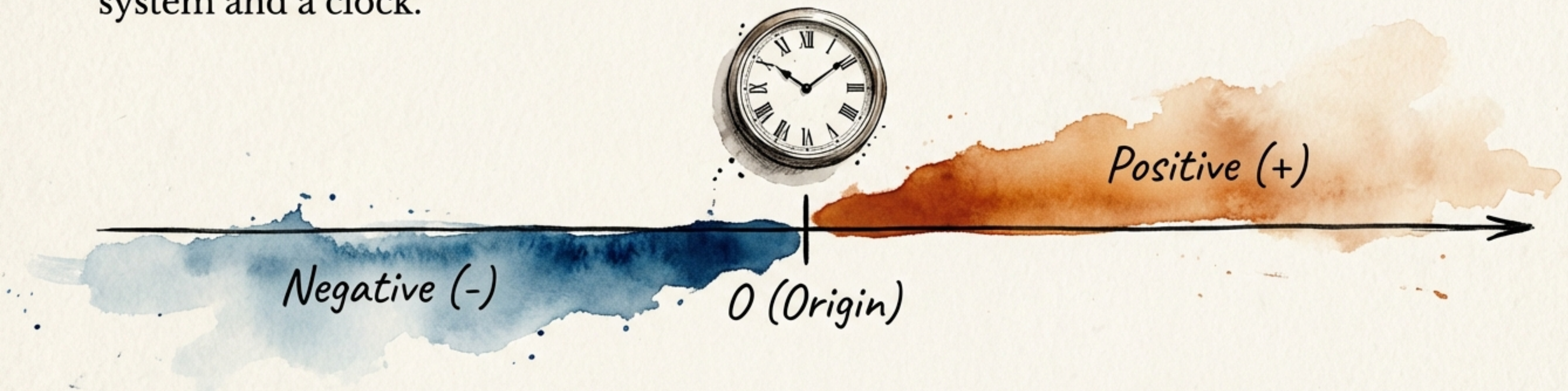
Defining the Canvas: Frame of Reference

Motion is a change in position with time. To describe it, we need a coordinate system and a clock.



Rectilinear Motion:
Motion confined to a straight line (the x-axis).

The Origin:
The reference point (x=0) from which all position is measured.

Time (t):
The variable that drives the flow along the axis.

Note: The choice of origin and positive direction is arbitrary, but must be defined before starting.

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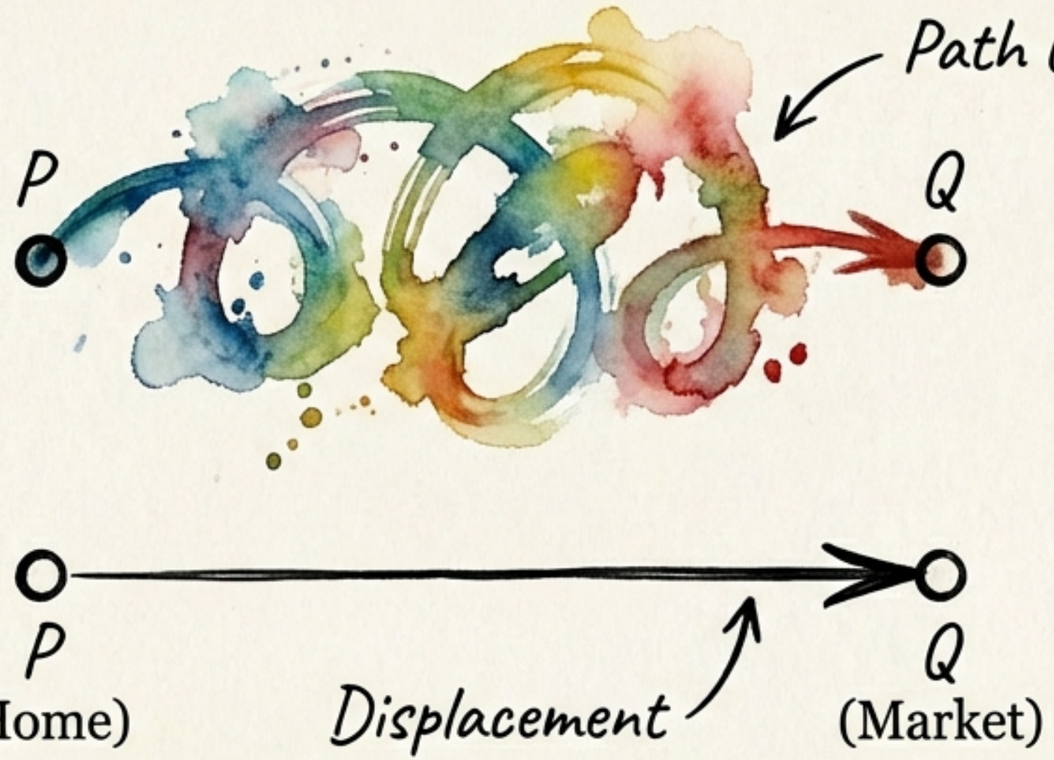
The Stroke vs. The Result: Distance and Displacement



Path Length (Scalar)
The total distance traversed. Like the total amount of paint used on the canvas. It is always positive.

Displacement (Vector)
"The shortest change in position ($\Delta x = x_2 - x_1$). It has both magnitude and direction. It can be positive, negative, or zero.

Example: A trip from Home to Market (2.5 km) and back Home.
Path Length: 5.0 km
Displacement: 0 km

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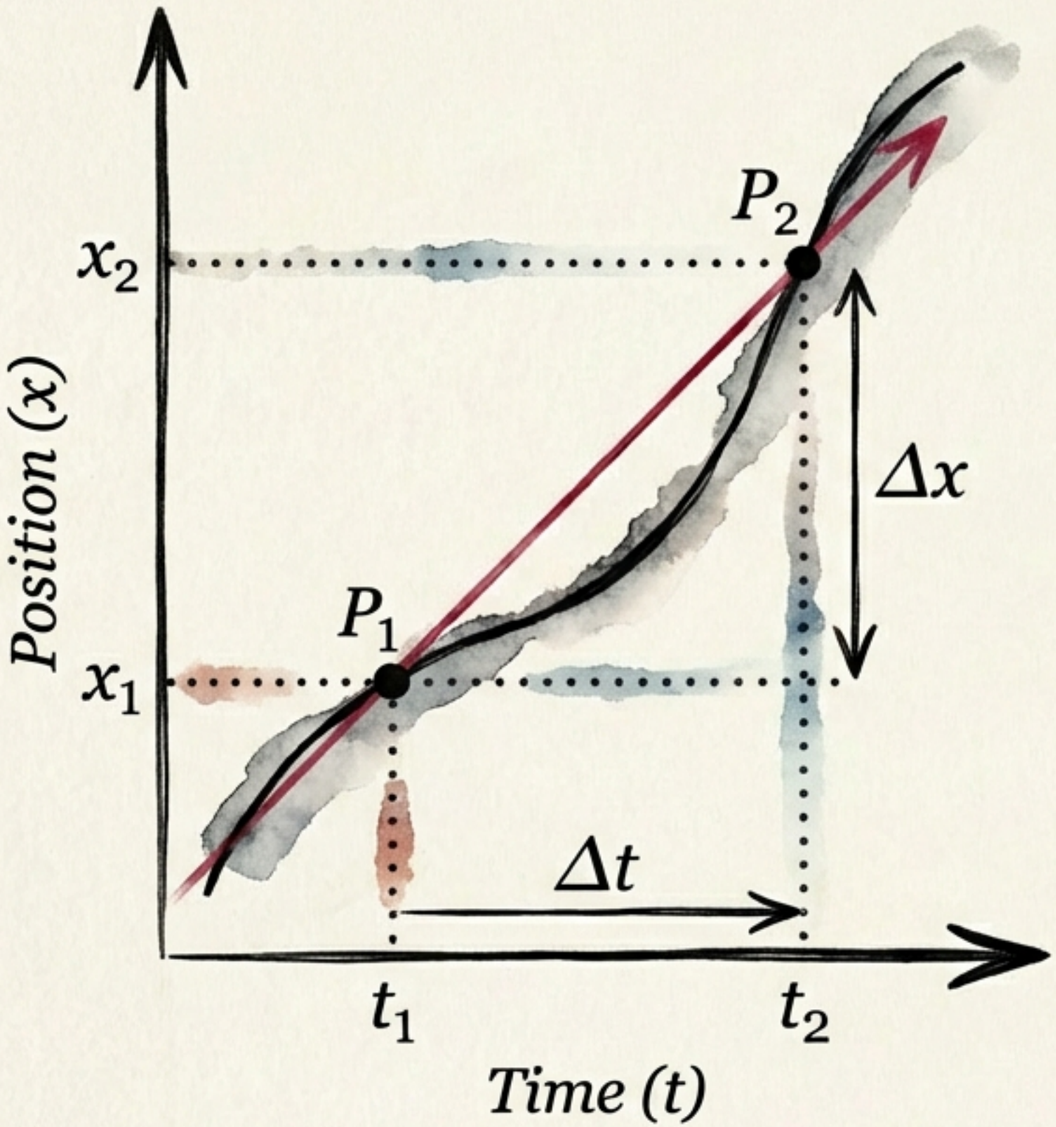

The Rate of Flow: Average Velocity

Average velocity describes how fast an object moves over a specific time interval, smoothing over the details of the motion.

$$\text{average } v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Distinction:
Average Velocity: Displacement / Time Interval (Vector)
Average Speed: Total Path Length / Time Interval (Scalar)
Note: Average speed is always ≥ magnitude of average velocity.

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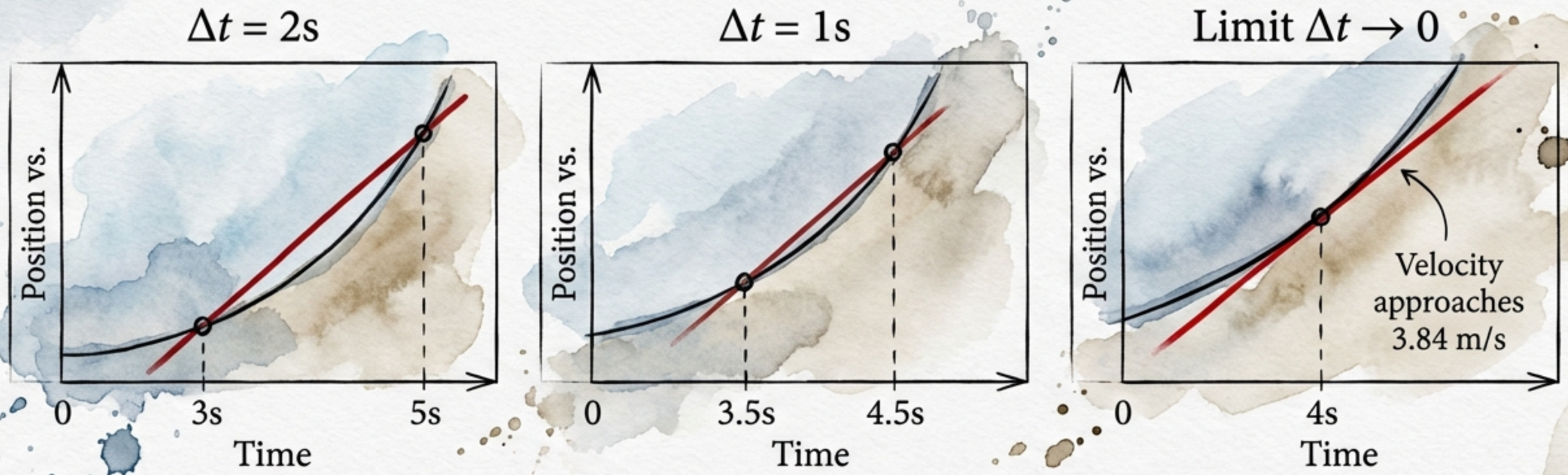
The Instant: Zooming In on the Canvas



Instantaneous velocity is the limit of average velocity as the time interval shrinks to zero. On a graph, this is the slope of the tangent.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

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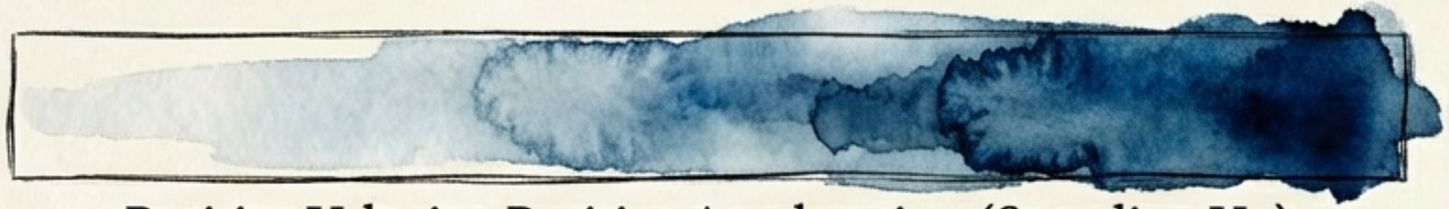
Changing Pressure: Acceleration



Positive Velocity, Positive Acceleration (Speeding Up)

Positive Velocity, Negative Acceleration (Slowing Down)

Negative Velocity, Negative Acceleration (Speeding Up)

Negative Velocity, Positive Acceleration (Slowing Down)

Acceleration is the rate of change of velocity: $a = \frac{dv}{dt}$.

Crucial Insight: The sign of acceleration alone does not define speeding up or slowing down.

- If ‘a’ and ‘v’ have the **same sign**, speed increases.
- If ‘a’ and ‘v’ have **opposite signs**, speed decreases.

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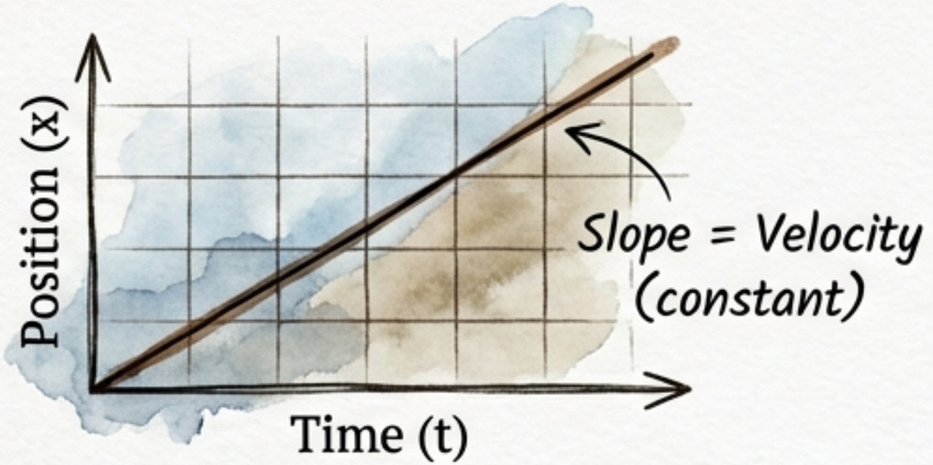
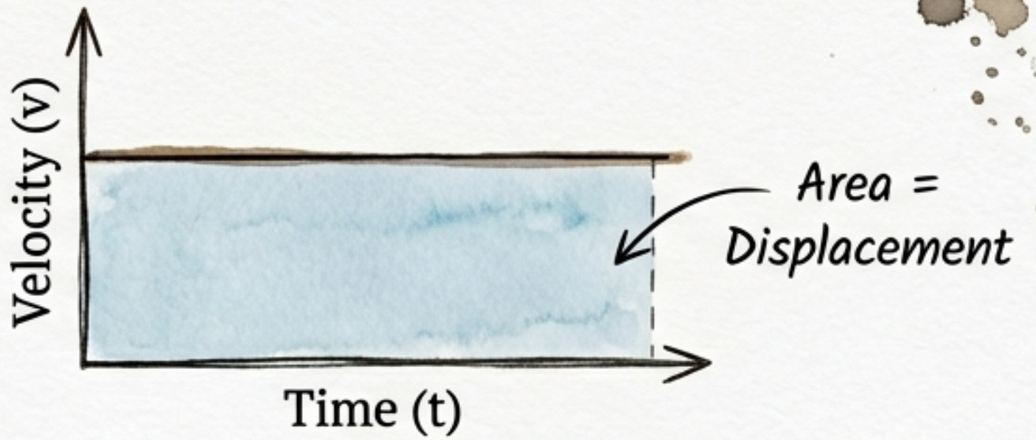
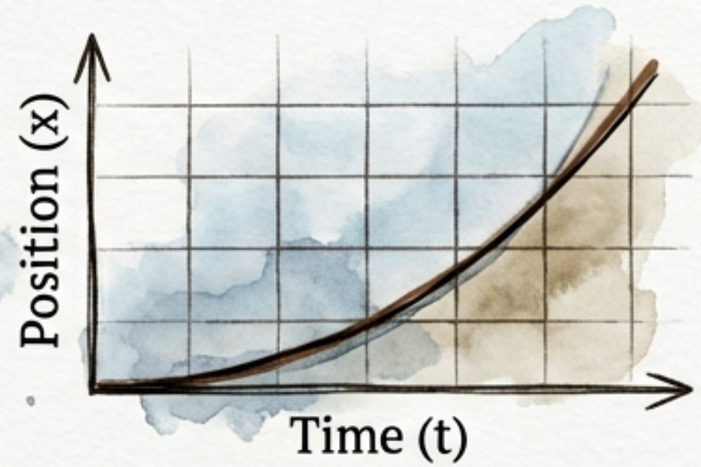
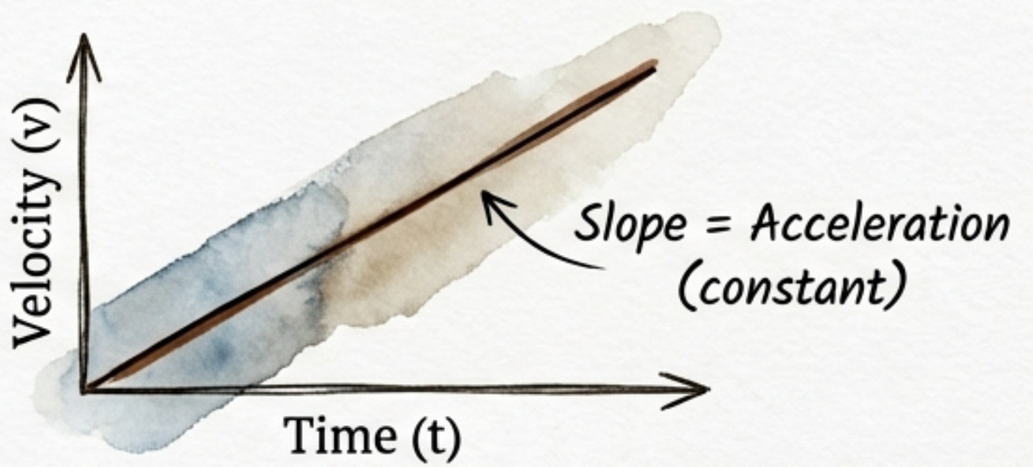
The Geometry of Motion









Interpreting the Slopes:

- The slope of the ***x-t* graph** gives Velocity.
- The slope of the ***v-t* graph** gives Acceleration.

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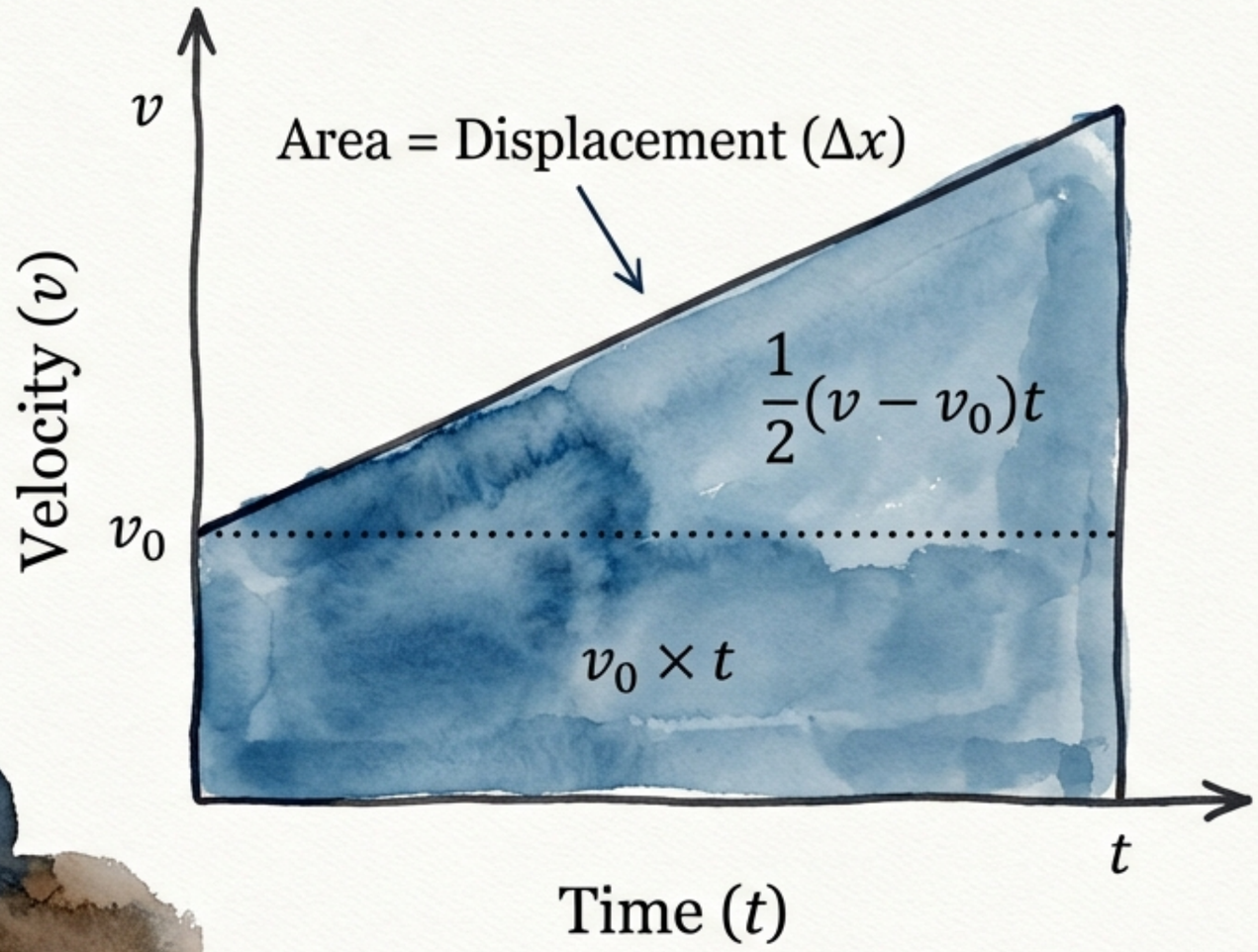
Accumulation: Area Under the Curve



The area under the velocity-time curve represents the **displacement** (Δx) accumulated over time.

For constant acceleration, this area is the sum of a rectangle and a triangle:

Area = (Initial Velocity × Time) + (Area of Triangle)

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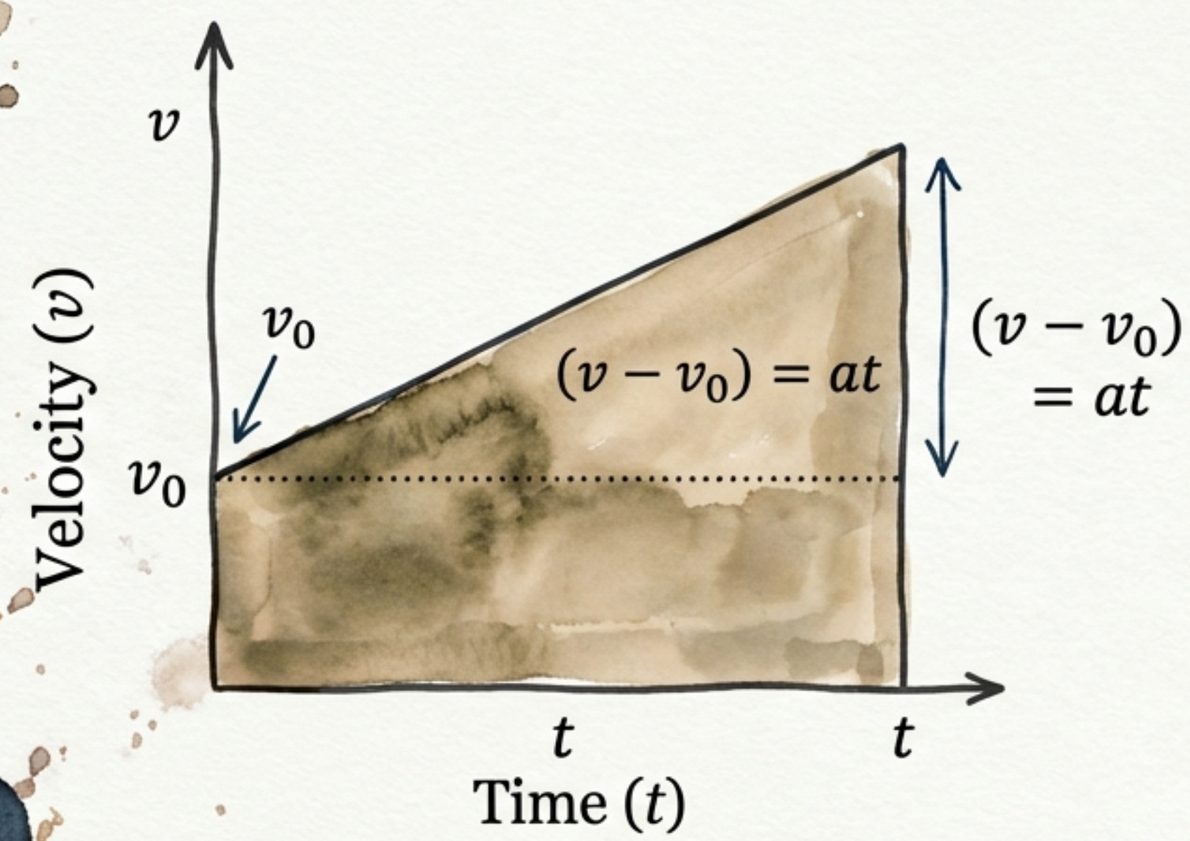
Deriving the Rules (Kinematic Equations)



1. We know from the slope that:
$$v - v_0 = at$$
2. Displacement (x) is the total area of the trapezoid.
3. Total Area = Area of Rectangle + Area + Area of Triangle
4. $x = (v_0 \times t) + \frac{1}{2} \times t \times (at)$

$$\boldsymbol{x = v_0 t + \frac{1}{2} a t^2}$$

This is simply geometry applied to the flow of time.

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The Kinematic Equations

(For Uniform Acceleration Only)

Velocity-Time Relation:

$$v = v_0 + at$$

Position-Time Relation:

$$x = v_0 t + \frac{1}{2} a t^2$$

Position-Velocity Relation:

$$v^2 = v_0^2 + 2ax$$

v_0 = Initial Velocity
a = Acceleration

v = Final Velocity
a = Acceleration (Constant)
x = Displacement

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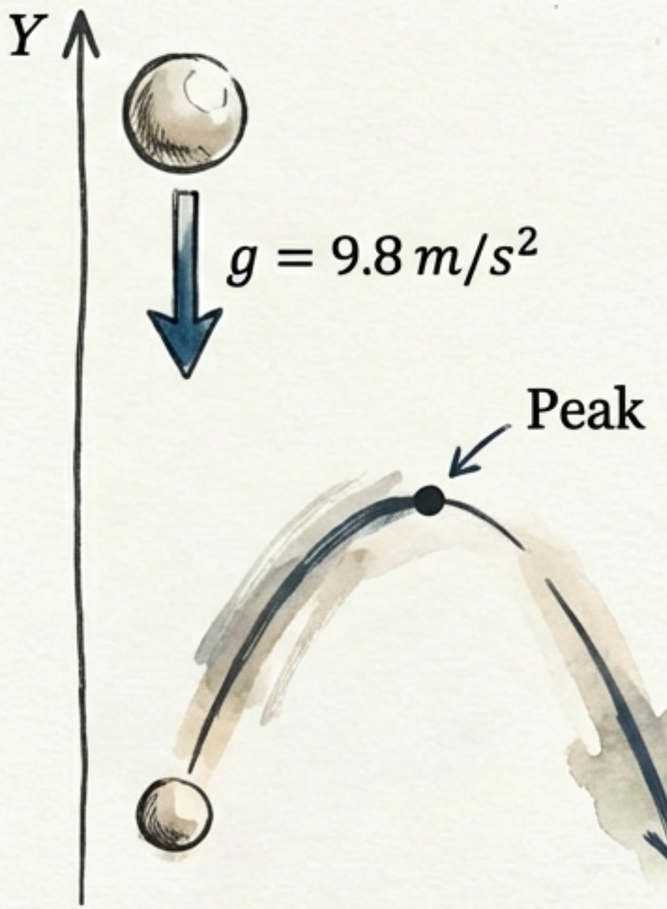
Nature's Constant: Free Fall

An object released near Earth falls under the influence of gravity alone (neglecting air resistance).

The Parameters:

- Acceleration $a = -g = -9.8\ m/s^2$
- The negative sign indicates the downward direction.



Common Misconception:

Caveat: At the very top of its path, the ball has zero velocity, but its acceleration is still -9.8 m/s². Gravity does not switch off just because the object stops for an instant.

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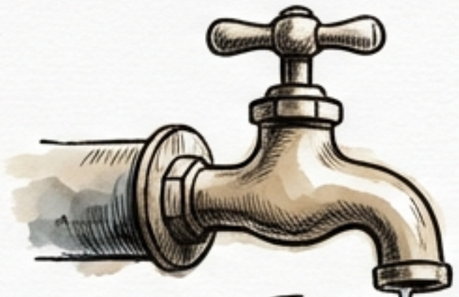
The Rhythm of Gravity: Galileo's Law of Odd Numbers



Interval 1: Distance d → 1

Interval 2: Distance $3d$ → 3

5

Interval 3: Distance $5d$ → 7

For a body falling from rest, the distance traveled in equal time intervals follows a harmonic pattern proportional to odd numbers.

Ratio of distances: 1 : 3 : 5 : 7 : ...

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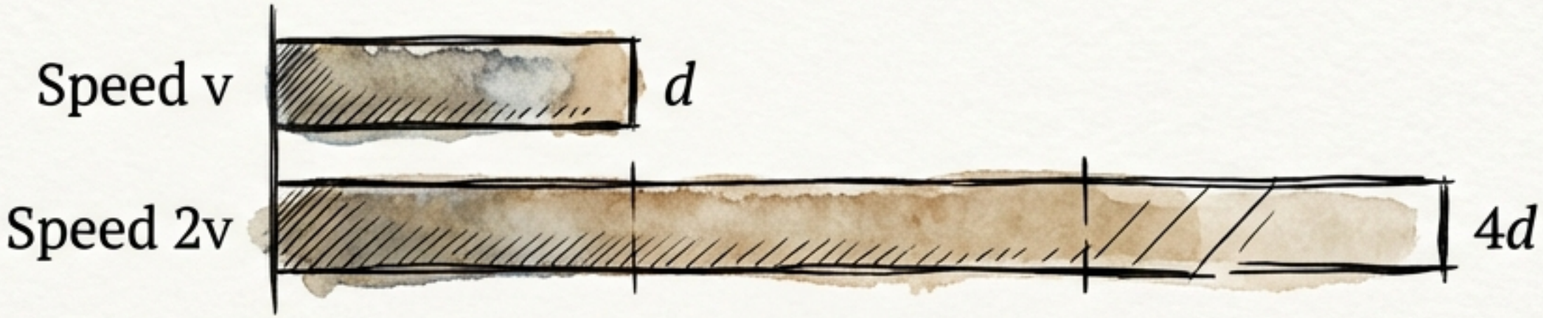
The Cost of Speed: Stopping Distance



Using the equation $v^2 = v_0^2 + 2ax$ (where final v=0), we find that stopping distance is proportional to the square of the initial velocity.

$$d_s \propto v_0^2$$

Safety Implication: Doubling your speed (2x) multiplies your stopping distance by four (4x).

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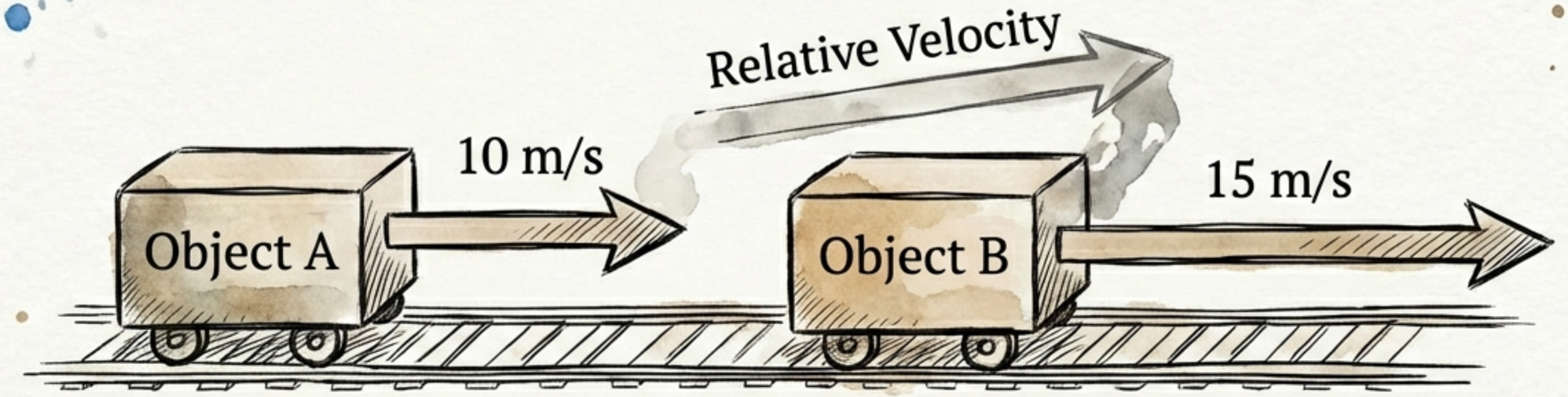
Shifting Perspective: Relative Velocity

Velocity is always measured relative to a frame of reference.
If the observer is also moving, we must subtract their velocity.



$$v_{BA} = v_B - v_A$$

(Velocity of B relative to A)

If you (A) move at 10 m/s and a train (B) moves at 15 m/s in the same direction, the train appears to drift past you at only 5 m/s.

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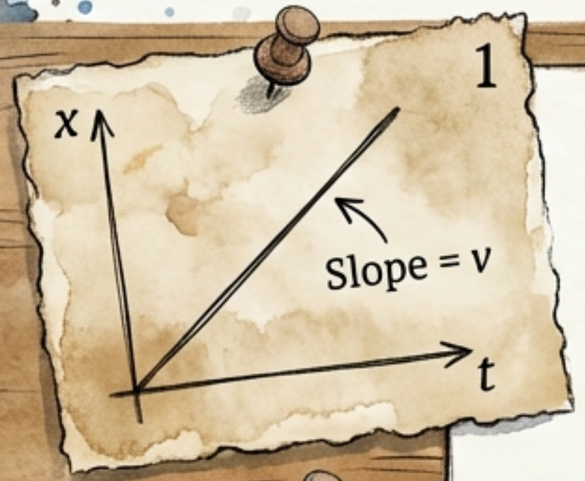
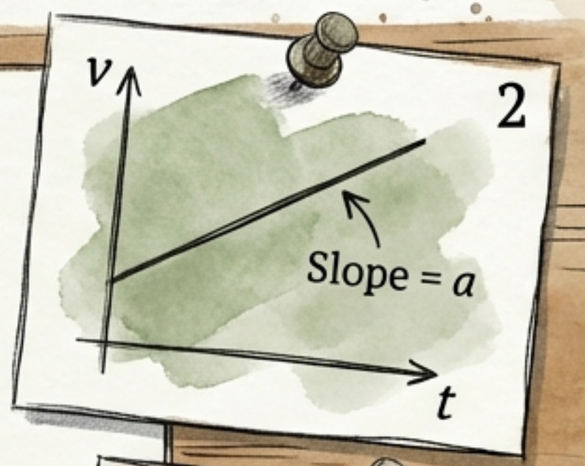
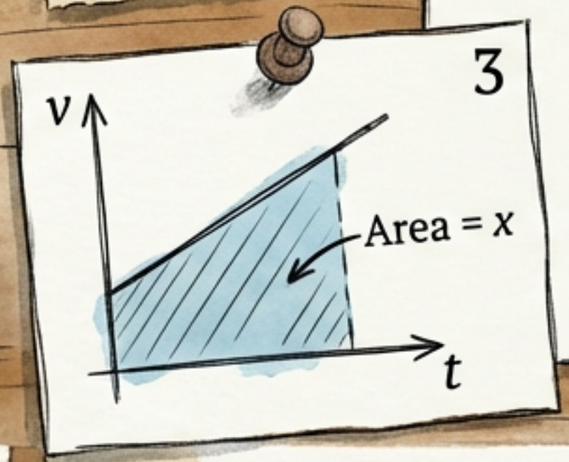
The Portfolio: Summary of Concepts







1. **Velocity (v):** The slope of the position-time graph.
2. **Acceleration (a):** The slope of the velocity-time graph.
3. **Displacement (x):** The area under the velocity-time graph.
4. **Kinematic Equations:** Tools valid only for uniform acceleration.

Kinematic Equations 4

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2ax$$

*The definitions of instantaneous velocity and acceleration are **exact** and always **correct**. The kinematic equations are tools for the special (but common) case of uniform acceleration.*

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